# World-Class Math Proficiency by Third Grade



Plug math into conversations with children.

**A Family Math Newsletter** 

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## Issue 1: Make Friends with Numbers

During this period of social-distancing, we can reach out to our old friends by phone, on-line platforms, and written correspondence, but it's difficult to meet new human friends. So, I encourage adults and children who are staying home from work and school to use this time as an opportunity to help children make friends with numbers. Goal #1 on my Top-Ten List indicates that math-proficient third graders should have skill fluency in ordering numbers from 1 to 1000. Now, this doesn't mean just recognizing them and memorizing their names in a list but means creating life-long friendships with these numbers. Whoa! One thousand friends! That's a lot of names to add to a list of contacts, but let's start smaller. Infants and toddlers (baby mathematicians) can begin making friends with numbers up to 10. Preschoolers, kindergartners, older children, and adults can deepen number friendships (strengthening those little connective synapses<sup>1</sup>) with the numbers from 1 to 100.

Acquaint Baby Mathematicians with 10 New Friends As born-again mathematicians, we can use our math lenses to zoom-out and gain an overview of the number-order-to-1000 learning trajectory or zoom-in on counting to 10 or 100. The infant and toddler years are perfect times to initiate one-way conversations about numbers from 1 to 10. Quantify things you see around you, "I see 2 cars in our neighbor's driveway," or "I see 4 cookies on the plate." Any time, any place, you can let the child hear you recite the number sequence and watch you count collections of objects. Whatever you choose enumerate, act like you're having fun. Be overly-dramatic and make up songs. Then, when your toddler begins to use language, encourage recitation of the count sequence. From my math-lens perspective, I believe learning the count sequence to be more important than the alphabet sequence. For example, with the letter "B", you must put it together with other

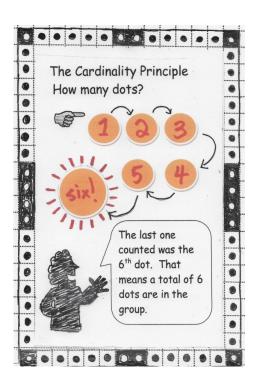
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<sup>&</sup>lt;sup>1</sup> Remember about synapses from Issue 0....When we learn new ideas, electric currents fire in our brains creating physical connections called "synapses." If we visit a new idea only once or in a superficial way, then the synaptic connections will soon disappear. However, if we learn something deeply, revisiting the idea periodically over several years, the synaptic activity will create lasting connections and form structural pathways. (Jo Boaler, *Mathematical Mindsets*, 2016)

letters to give it meaning, but the number 6 has meaning all its own. Recorded songs and videos about counting and numbers can also be helpful.

The Cardinality Principle As a child is learning the number sequence up to 10 and higher, then it's time to help her learn how to count collections. Counting a collection of objects provides practice with motor skills developing coordination between touching the object and saying the number's name ("one" - "two" - "three"). Furthermore, as you help children count small collections, use your math lenses to zoom in on the super-important cardinality principle -- the knowledge that the number name associated with the last object counted reveals the total number of objects in the set. Understanding this big idea transforms a counting process to a result, so review it repeatedly. For example, a child can count 5 items and say "That makes 5." Then, keeping this result in mind, she can count 5 more objects and conclude there are 10 all together. If you will describe the cardinality principle repeatedly to a child as you count things with her, then she will understand it before turning four.



Three (3) Power-Chats To promote deep understandings of the numbers from 1 to 100, here are three activities you might try out within your family. Study them yourself first and think of ways you can use aspects of them to engage different age groups. Use your math lenses to connect them with everyday activities, picture books, and what your children already know.

#### Make Friends with Numbers

- Power-Chat 1: Meet 100 New Friends Today!
- Power-Chat 2: The Idea of 10 as a Unit
- Power-Chat 3: What's Your Favorite Number?

But before reading through the first activity, let me explain what I mean by a "power-chat." Not just any verbal exchange about numbers is necessarily powerful, particularly if the conversation merely reviews what the child already knows. A power-chat should raise the child's level of thinking, but should not be on a level that is too high.<sup>2</sup> Connecting with a child's interests can also give conversations power-chat qualities. Power-chat interactions should be enjoyable for children and adults. Participants encourage others to share thoughts and listen carefully to each other.

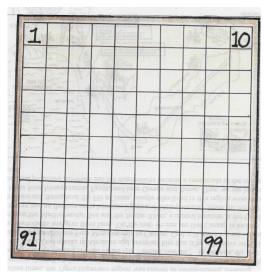
A true power-chat should be engaging, meaningful, and yet, have high expectations embedded in it. Children become engaged in the conversation if they find the topic interesting and worthwhile. If engagement is not present, then this may be the signal to try a new approach or move to a different activity. If children seem particularly interested, this

<sup>&</sup>lt;sup>2</sup> To me, a power-chat can be likened to a **Vygotsky's idea – "Zone of Proximal Development**." Google it to learn more. © Amanda Neel Davis, Ph.D.

may be a signal to go higher up the learning trajectory. For Goal #1, a move to the next level might involve considering numbers between 101 and 200; 201 and 300; etc.

# Power-Chat 1: Meet One Hundred New Friends Today!

To prepare for Power-Chat 1, construct a 100-grid from poster paper, cardboard, or foam board. You can see that my sample grid contains only four numbers. Use these numbers to set up the structure for writing the other 96 numbers on the table.<sup>3</sup> The general plan will be to measure with a yard stick (36 inches or 91.44cm) or a meter stick (100 centimeters or 39.37inches) and construct a large square containing 100 small squares in 10 rows and 10 columns. If using cardboard or foam board, make two halves so that can it can be taped together and folded.



The table-construction activity is a good one for older children, as it is a project which hones measuring skills and builds proficiency for Goal #8.<sup>4</sup> Here are more detailed directions for three sizes of a 100-grid table. First, use a pencil so that you may erase and correct mistakes before completing the blank 100-square table with a marker.

## Small 100-grid (Poster paper 22"by 28")

- 50cm by 50cm square grid; 100 squares 5cm on a side
- 20" by 20" square grid; 100 squares 2 inches on a side

#### Medium (cardboard or foam board)

- 80cm by 80cm square grid; 100 squares 8cm on a side
- 30" by 30" square grid; 100 squares 3 inches on a side

#### Large (cardboard or foam board)

- 100 cm by 100 cm square grid; 100 squares 10 cm on a side.
  - 40" by 40" square grid; 100 squares 4 inches on a side.

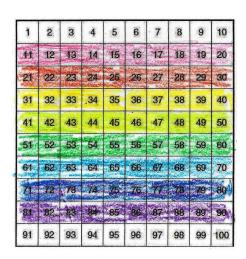
Writing the numbers on the table is a good grade-schooler task (if grade-schoolers are available). Again, use a pencil first so that mistakes can be corrected. Then, complete the table with black-marker numerals. After the numbers are on the 100-grid, children can color in the backgrounds for the squares. Before passing out the crayons, collect some ideas from children about a plan. One idea for a plan could be to make a "rainbow 100-grid" coloring each row or column all one color (as pictured). Or, perhaps use a color design similar to a tiling pattern. Another idea is to divide the table into quarters by coloring 25

<sup>&</sup>lt;sup>3</sup> Starting with 1 and ending with 100 is not "set in stone" but a choice. We could have started with 0 and ended with 99. Measuring devices actually start at zero.

<sup>&</sup>lt;sup>4</sup> Goal # 8:Measure, compare, and estimate lengths with commonly used measuring devices scaled in inches, feet, yards, centimeters, and meters.

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squares the same color. This coloring task could easily be assigned to four children. The children may decide on a more free-form color design.



After your 100-grid visual aid is completed, lay it down on the floor for a very physical warm-up activity. Have everyone to stand in a circle around it, jump up and down, and count loudly to 100 by ones. Consider adding motions to this verbal count, changing the motion each time you complete a decade. After counting to 100 by ones, everyone should give a cheer.<sup>5</sup> After the cheer, count to 100 by tens (the quick way). Now, post the 100-grid where everyone can see it and begin the conversation.

Where is forty-three? First do a little orientation for navigational purposes. Locate the numbers 1 through 10. Then, locate smallest number; the largest number and

middle numbers on the table. Talk about the meaning of "row" and "column." <sup>6</sup> Ask if anyone wants to talk about something observed on the table. Then, ask if any of the younger children can find 43. Give some hints if need be. Associate the number's numeral-name with the word "forty-three" (both written and oral). Now, use the 100-grid table to answer the following questions about forty-three.

- 1. What number comes just before 43? And just after it?
- 2. What number is just below 43 on the 100-grid table? What number is just above it?
- 3. On what number do you land if you count forward 3 from 43 and backward 2 from 43 (reminiscent of a playing a board game). On what number do you land if your count forward 5 from 43 and backward 3 (up to 40 on the previous row). Clarify which direction is forward and which is backward. (At this point, adults might notice that a 100-grid is read from left to right and top to bottom, just like pages of a book.)
- 4. Now look at the column containing 43. Is it the 1<sup>st</sup>; 2<sup>nd</sup>; 3<sup>rd</sup>; 4<sup>th</sup>; etc.? Ask what pattern connects 43 with the 3<sup>rd</sup> column? (algebraic thinking)
- 5. Notice the row containing 43 is the 5<sup>th</sup> row, what's the algebraic pattern for this relationship? The rule for numbering rows will be different but algebraic thinking will still be employed.

If children seem interested, have them select another number to power-chat about. Leave the 100-grid in a place where people in the family can view it, interact with it repeatedly, and ask questions. Here's another idea... Post both a calendar and a 100-grid close together. Then you can look for calendar dates on the 100-grid and numbers between 1 and 100 on the calendar. Ask children to select numbers and then talk about whether or not their selected number is a possible calendar date.

<sup>&</sup>lt;sup>5</sup> Children will enjoy repeating this count to 100 as a warm-up activity. Encourage them to make up their own special cheer.

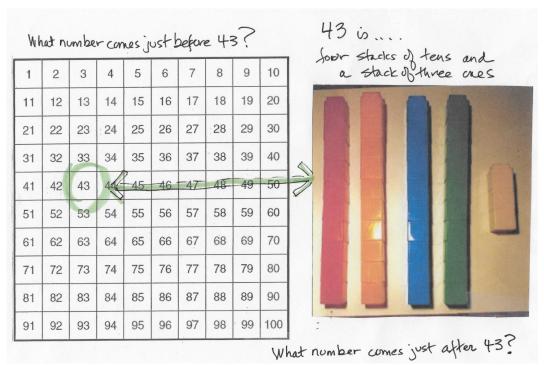
<sup>&</sup>lt;sup>6</sup> Here's a first spreadsheet lesson for preschoolers and kindergartners.

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## Power-Chat 2: The Idea of Ten as a Unit

"A critical achievement in a child's mathematical life is the idea of 10 as a unit."

Making true friends with a number not only involves recognizing the number and finding it on the grid but also having a sense of its size by considering the place-values of its digits. Let's go back to our good friend 43. We expand on the cardinality principle when we look at 4 sticks composed of 10 stacking cubes and say, "That's 10,20,30,40." Then, if we stack up 3 more cubes, the total count is 43. Stacking cubes can be used to create a visual representation of a number's value that can be quickly interpreted. The diagram below shows connection between a concrete representation of the number 43 as four tens and three ones with the numeral 43 – a 4 in the tens place and a 3 in the ones or units place. The numeral 43 is a symbolic representation of the same number represented concretely using stacking cubes.



Now, have children use stacking cubes to apply the 10-as-a-unit idea and represent other numbers on the 100-grid. Children become engaged in grouping into 10s because this kind of thinking makes navigating a 100-grid into a much shorter process than counting to 100 by ones. grouping by 10s is a much shorter process than counting by ones. Then, when a first or second grader looks at a 100 grid, he will see the cardinality of 53 or 63 by adding one or two more stacks of 10.

<sup>&</sup>lt;sup>7</sup> This is a quotation of Leslie P. Steffe, Professor Emeritus of Mathematics Education at the University of Georgia, who has inspired me in my work. He named the idea of ten as a unit as the first, out of three, critical achievements in a child's mathematical life. Number-two achievement was the idea of a fraction and the 3<sup>rd</sup> achievement was the idea of an unknown.

 $<sup>^{\</sup>rm 8}$  Stacking cubes, also known as "Unifix Cubes" can be purchased on the Internet.

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#### A Hundred Pennies Make One Dollar

Another way to visualize the 10-as-a-unit concept is by using coins. Now let's go back again to the number 43. Pictured are 4 glass cups, each containing 10 pennies, and 3 more pennies just to the right of the four cups. Just as with stacking cubes, we can "see 43" without counting by using the 10-as-a unit concept. One quick glance tells us we have 10, 20, 30, 40 -- 43 pennies in all which makes 43¢. If you have 100 pennies on hand, you can

give children experiences grouping in tens to show the value of other numbers. We also unitize by tens when we count dimes 10, 20, 30, 40, ... \$1). Other grouping-in-tens activities include bundling popsicle sticks in groups of 10 and placing rubber bands around each group. Also, count out groups of 10 snack crackers, buttons, or other small items. Place them in egg cartons, small cups, trays from candy boxes, etc.

When counting money, we can also unitize by 5s when counting nickels (5, 10, 15, .... \$1) and unitize by 25s when counting quarters (25, 50, 75, \$1, 125, 150, 175, \$2).



## Power-Chat 3: What's Your Favorite Number?

In this activity, children will ask family members and friends to select one favorite number from 1 to 100. When you introduce this favorite-number task, talk to children about

Person's Name	Favorite Number			

how it's okay to treat our number friends differently from our people friends. With people, we must be careful not to hurt feelings, but numbers don't get their feelings hurt if you choose another number as the favorite. Assure children that choosing 58 won't offend 48 or 78. Also, assure children that tomorrow they can keep the same favorite number, choose a different one, or have multiple favorite numbers. Begin by having each child make his own data collection sheet like the one pictured here. Older children and adults can help (and possibly record for younger children). After recording answers to the favorite-

number question for everyone in the home, have a discussion.

- Why did you choose \_\_\_\_ as your favorite number?
- Did you notice preferences for smaller or larger numbers?
- Were there certain numbers that were particularly popular?

Next, have children phone family members and friends and collect more data about favorite numbers. Ask friends why they chose a particular number and share reasons for your own choice. To complete data collection part of the activity, combine individual data collection tables into one master table. Depending on children's sustained interest, use your own judgment about how many people to ask and how many data points to have in the table. And, it wouldn't be a bad idea to put your master table in alphabetical order.

## Data Organization and Analysis

Now that the data is collected, we will organize and analyze it, first by making an organizational table and then by creating a bar graph. However, before making this table you need to decide how many consecutive number intervals you want to have. Of course, there are 100 possible choices for favorite numbers, but having 100 rows in your tally table or 100 bars on your graph is unworkable. Thus, I present a workable plan of placing favorite numbers into 5 intervals, each containing 20 numbers. As an intermediate activity to illustrate how the choice of 5 intervals might work, download a master copy of a 100 grid off the internet and print copies for children to color. Give them an example of a grid with 5 stripes like the one shown. Each stripe will contain 20

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71-	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

number squares. The stripes should be at least two different colors. Perhaps suggest that children use school colors or colors of a favorite sports team. A choice of 5 favorite colors would also work. But use crayons, colored pencils, or highlight markers so that the numbers will be readable.

#### Complete a Data Organization Table

Data can be organized by reading down the names in the list and placing tally marks in the correct interval for each one. One child or adult can read out names and favorite numbers and another enter a tally mark in the appropriate row. After making the table talk about the results. Start with the children's ideas but here are some additional analysis points.

- What was the range containing the most favorites? (1-20? 21-41? etc.)
- Did people prefer high numbers, low numbers or mid-range numbers?
- Was there one particular number that was the most frequently selected?
- Think of additional discussion points based on children's ideas.

Data Organization	Tally	Number
Table	Marks	
1-20		
21-40		
41-60		
61-80		
81-100		
	total	

## Construct a Histogram Graph

If any of the children still seem engaged show them how to create a histogram to further analyze the data and think more deeply about results of this favorite number project. Another idea would be to save the graphing of the data for another day. The statistical process we used to create tables and graphs for collecting, organizing, and analyzing our data in the favorite-number activity is basically the same standard process used by statisticians. The following example provides a reference.

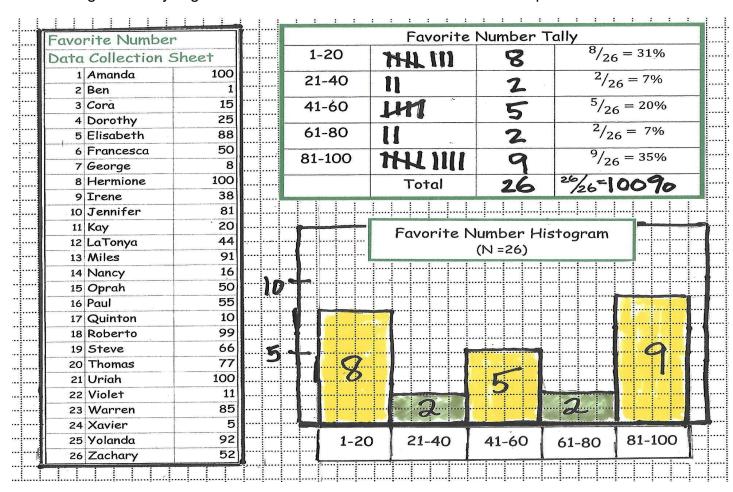
<sup>&</sup>lt;sup>9</sup> A histogram is a form of bar graph in which categories are consecutive equal intervals along a numeric scale. © Amanda Neel Davis, Ph.D.

## An Example of a Statistical Process

In 2005, the American Statistical Association (ASA) produced a report which acts as a set of standards for teaching statistics on academic levels A, B, and C covering prekindergarten through twelfth grade. An important part of these guidelines is the list of four steps:

- 1. Formulate Questions
- 2. Collect Data
- 3. Analyze Data
- 4. Interpret Results

The ASA recommends the use of these four essential steps on every teaching level – from preschool to university and beyond.<sup>11</sup> The diagram below provides sample tools for collecting and analyzing the data – tools which will then be used to interpret results.



There are truly unlimited real-world contexts for which the basic statistical process used in our favorite number study is applicable. As a follow up activity, you could use the names of the 26 people on my data collection sheet to consider the question, "How many letters are in the average person's first name?"

<sup>&</sup>lt;sup>10</sup> Guidelines for Assessment and Instruction in Statistical Education (Franklin, et al., 2005) is known as the GAISE Report. Level A statistics standards guide preschool through the elementary grades; Level B standards generally apply to middle school; and Level C specifies high school standards.

<sup>&</sup>lt;sup>11</sup> The GAISE Report for college students is a separate report, which was updated in 2016.

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# Closing Thoughts

At the end of each issue, I plan to summarize by locating the power-chats and other activities along a learning trajectory map relevant to the featured goal (Goal #1). Other goals related to the activities were Goals #6, #7, and #8.



## Goal #1: A Learning Trajectory

Order and compare numbers from 1 up to 1000. Understand place value, including ones, tens, hundreds, and thousands. Interpret, use, and construct visual and mental representations of number order, including 100-grids and number lines.

**Kindergarten Sub-Goals**: Initiate conversations about counting to 100 by 1s and 10s. Provide children with experiences using a 100-grid as a visual representation of number order. Locate numbers on the grid and develop place-value understandings, using concrete materials.

#### Birth/ Age 3

Counting can be used to find out how many things are in a collection.

- Count sequence
- Coordination(touching & saying number name)
- Cardinality principle

#### Preschool/Kindergarten

- Row/column tables (Power-Chat 1)
- Locating numbers on 100grid (Power-Chat 1)
- Moving forward & backward on the 100-grid (Power-Chat 1)
- Using the idea of 10 as a unit to determine a number's value (Power-Chat 2)
- Counting money (Power-Chat 2; Goal #8)

## Kindergarten/ Grades 1 through 3 and up

- Measuring in inches or centimeters to construct the 100-grid table (Power-chat 1; Goal #7)
- Unitizing as part of an efficient counting process (Power-Chat 2)
- Statistical thinking: 4 steps of a statistical process (Power-Chat 3; Goal #6)

## Additional Resources

There are many internet resources for learning all kinds of fun and interesting things about the numbers from 1 to 100. In addition, let me recommend two books and a website as general resources about doing math within the family.

• Family Math (1986) and Family Math for Young Children (1997) contain a wealth of activities which can help family members rediscover the fun and excitement in making math discoveries together, uniting people of all ages in the process of learning from each other. Although published more than 20 years ago by the University of California, Berkeley, these two books authored by Jean Kerr Stenmark and others, may be purchased new or used on Amazon, ThriftBooks, and Ebay websites.

• **Bedtime Math** (<a href="http://bedtimemath.org/">http://bedtimemath.org/</a>) The following statement appears on the home page: "Our charitable mission is to help kids love numbers so they can handle math in real life." The extensive resources available on this website include free apps, games, daily activities as well as recommendations for children's books.

Issue 2 "Multiplication Tables by the End of Third Grade" Next, we'll think about Goal #2 (knowing from memory all one-digit addition and multiplication facts). In Issue 2, I plan to skip past addition facts (which will be emphasized in Issue 3 on problem solving) and focus on learning the multiplication facts by the end of third grade. I truly believe that quick recall of multiplication facts can be a game-changer for a child's educational future. Even so, we should always remain aware that facts memorized quickly – just before the end-of-third-grade math achievement test – may not be retained over the long term. By starting early and guiding preschoolers up the learning trajectory of multiplicative thinking, third graders will find it easy to recall one-digit multiplication facts quickly. Then, in the future, they can use them to compute fluently and apply them in solving problems.

Please contact me if you have questions or comments. Also, send me Email addresses of others who might be interested in this newsletter.

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